Dr. Usha Kumari

Sem II MJC Unit 2.

Resultant of Two S.H.M.s at Right Angles

(Same period but Differing in Amplitude and phase).

$$x = a\sin\omega t \quad \text{--- (1)}$$
$$y = b\sin\omega t + \phi \quad \text{--- (2)}$$

$$\sin\omega t = \frac{x}{a}$$

$$\therefore \cos\omega t = \sqrt{1-\frac{x^2}{a^2}}$$

$$\frac{y}{b} = \sin(\omega t+\phi)$$

$$= \sin\omega t \times \cos\phi + \cos\omega t\sin\phi$$

$$\therefore \frac{y}{b} = \frac{x}{a}\cos\phi + \sqrt{\left(1-\frac{x^2}{a^2}\right)}\sin\phi$$

$$\Rightarrow \left(\frac{y}{b}-\frac{x}{a}\cos\phi\right) = \sqrt{\left(1-\frac{x^2}{a^2}\right)}\cdot\sin\phi$$

squaring both sides

$$\Rightarrow \left(\frac{y}{b}-\frac{x}{a}\cos\phi\right)^2 = \left(1-\frac{x^2}{a^2}\right)\cdot\sin^2\phi$$

$$\Rightarrow \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2}\cos^2\phi + \frac{x^2}{a^2}\sin^2\phi - \sin^2\phi = 0$$

$$\Rightarrow \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2}(\cos^2\phi + \sin^2\phi) = \sin^2\phi$$

$$\therefore \boxed{\frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2} = \sin^2\phi} \qquad (3)$$

This represents the resultant motion of the particle, which in general an ellipse inclined to the axes of co-ordinates.

Important Cases: —

Case (i) when $\phi = 0^c$, then

$\sin\phi = 0$ and $\cos\phi = 1$

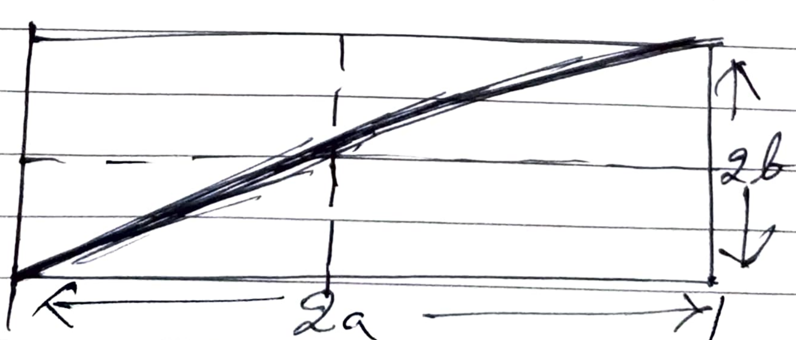
Then from equ^n (3)

$$\frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2} = 0$$

$$\Rightarrow \frac{y^2}{b^2} - \frac{2xy}{ab} + \frac{x^2}{a^2} = 0$$

$$\Rightarrow \left(\frac{y}{b} - \frac{x}{a}\right)^2 = 0$$

Thus, the resultant motion in a pair of coincident straight lines lying in quadrants I & III of ~~the~~ rectangles as shown in flowing fig. The straight lines are inclined to the x-axis at an angle θ given by $\theta = \tan^{-1}\left(\frac{b}{a}\right)$



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